Interest-Bearing Central Bank Digital Currency and Banking Redux

Adib Rahman*

Department of Economics and Finance, Neil Griffin College of Business, Arkansas State University

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Abstract

I develop a general equilibrium model to study the optimal design and implementation of central bank digital currencies (CBDCs) in an economy where CBDCs and private bank deposits coexist as competing payment instruments. The findings suggest that the welfare consequences of CBDCs and the policies required for first-best implementation depend on specific parameters. In sufficiently patient economies, a passive monetary policy with non-interest bearing CBDC can achieve the first-best allocation without crowding out bank deposits. Conversely, in more impatient economies, active policies with positive inflation and nominal interest rates are necessary, and interest-bearing CBDC could potentially crowd out bank deposits if the interest rate on CBDC is too high relative to deposit rates. The results highlight the importance of carefully designing CBDCs with incentive-feasible policies intended to maximize welfare and minimize risks to financial intermediation. In a calibrated model representing the US economy, the welfare consequences of interest-bearing CBDC are non-monotone in the CBDC interest rate: moderate positive rates improve welfare relative to a non-interest-bearing benchmark, while higher rates produce sufficient bank disintermediation to reduce welfare on net. The constrained welfare-maximizing CBDC nominal rate is active but moderate (approximately 3.4%), with deposit disintermediation of approximately 8% at this optimum. At the upper end of the admissible range (a 5% CBDC nominal rate), disintermediation reaches approximately 12%.

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*Department of Economics and Finance, Neil Griffin College of Business, Arkansas State University, Jonesboro, AR 72467, USA. E-mail: arahman@astate.edu.

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1 Introduction

Central bank digital currencies (CBDCs) have emerged as a topic of significant interest among policy makers and economists in recent years. The potential benefits and risks of CBDCs have been widely discussed, with many central banks actively exploring the possibility of issuing their own digital currencies.[1] Major economies worldwide are actively researching CBDCs, which would represent a third form of currency accessible to the public, akin to cash, and also accessible to many financial institutions, similar to central bank reserves.[2] Several central banks have conducted or are in the process of planning pilot programs, and operational CBDCs already exist in Caribbean-island countries, such as DCash in the Eastern Caribbean Currency Union (ECCU), the Sand Dollar in the Bahamas, and Jam-Dex in Jamaica. China stands out as a key player among populous nations, aiming to extend financial services across extensive sectors of its economy. Additionally, India and Indonesia are currently conducting trials for digital versions of their respective currencies, the rupee and the rupiah.[3]

A wide range of technological designs for CBDCs have been proposed, but a fundamental characteristic of a CBDC is that it must be universally accessible, meaning it can be held by anyone for any purpose. A second feature relates to whether CBDCs are interest-bearing. The interest rate can serve as an additional policy tool, expanding upon the existing monetary toolkit to stabilize inflation and output. One frequent policy concern surrounding CBDCs is their potential impact on the banking system and financial intermediation. Specifically, many economists and policymakers have expressed concerns about whether CBDCs could lead to a crowding out of bank deposits, as households substitute traditional bank accounts for holding CBDCs. This disintermediation effect could have significant implications for bank lending and investment, as well as for broader concerns about financial stability.[4]

In this paper, I develop a general equilibrium model to study the conditions under which the introduction of a CBDC could crowd out bank deposits and lead to a reduction in bank-financed investment. I build on the framework of Andolfatto et al. (2016) and the subsequent New Monetarist literature, by introducing a CBDC that competes with bank deposits as a means of payment. In the model, households can voluntarily choose their portfolio allocation between CBDC and bank deposits based on their relative returns and liquidity properties. Banks, in turn, make investment decisions based on the level of deposits they attract. I distinguish between

[1] According to Boar and Wehrli (2021), a survey conducted by the Bank for International Settlements involved 65 central banks. The survey revealed that 86% of these banks are actively engaged in initiatives related to CBDCs, with 60% having initiated experiments or proofs-of-concept for CBDCs. Additionally, 14% of the banks have progressed to the stage of developing and piloting CBDC arrangements.

[2] See Auer et al. (2020) for an overview of the policy discussion surrounding CBDCs and their different designs across a diverse mix of countries.

[3] Updated lists of countries investigating or issuing CBDCs are reported by `https://www.atlanticcouncil.org/cbdctracker/`.

[4] See Meaning et al. (2018) for a discussion on the potential impact of CBDC on monetary policy transmission and the risks CBDC poses to the banking sector.

the liquidity properties of bank deposits by introducing checkable deposits (more liquid) and time deposits (less liquid). A key feature of the model is that I explicitly incorporate the design choices around CBDC, such as the interest rate it pays and any fees associated with its use. This allows us to study how these policy parameters interact with household portfolio choices and banks' investment decisions. Importantly, the government is assumed to lack a lump-sum tax instrument to implement its monetary policy rule. This implies that households' currency choices must adhere to sequential rationality constraints, as there is no coercion or forced participation. Monetary policy, in this sense, must be incentive-feasible.

I focus my attention on implementing first-best allocations. The main results characterize the government policies required to implement the first-best allocation in an economy where CBDC and bank deposits coexist as payment instruments. In sufficiently patient economies, a passive policy with constant money supply is enough to achieve the first-best outcome. No inflation or taxes are needed, and CBDC does not crowd out bank deposits if CBDC is non-interest bearing. Banks are willing to issue deposits and pay interest rates to households at a sufficiently high level, as households are willing to sacrifice enough of their consumption for labor in the production of the output. Moreover, asset prices are priced at their fundamental level. There is no liquidity shortage as the consumer debt-constraints are slack.

In impatient economies, active policies with positive inflation and nominal interest rates are required to implement a first-best allocation with competing payment instruments. Binding debt-constraints lead to a shortage of liquidity. Banks issue fewer deposits to households and offer excessively high interest rates to motivate households to work hard and produce enough output. Liquidity premium arises due to asset scarcity as a result of the binding debt-constraints. In the absence of lump-sum taxes, the government must use some combination of seigniorage revenue, labor income taxes, and CBDC fees to incentivize efficient production by households and deposit issuance by banks. In this case, interest-bearing CBDC can potentially crowd out bank deposits if the interest rate on CBDC is too high relative to deposit rates. Distortionary taxes and CBDC fees are necessary to relax the debt-constraints of households.

To quantify these theoretical predictions, I calibrate the model to match key features of the US economy using data from 1987 to 2008. Consistent with the theoretical prediction that interest-bearing CBDC can crowd out bank deposits, the quantitative analysis shows that raising the nominal CBDC interest rate produces substantial bank disintermediation: at the constrained welfare-maximizing rate of 3.4%, bank deposits decline by approximately 8%, and at the upper end of the admissible range (a 5% CBDC rate), they decline by approximately 12%. The welfare effects, however, are non-monotone. At low positive CBDC rates the relaxation of household debt-constraints expands PM trade and improves welfare; at higher rates the disintermediation channel begins to dominate. The constrained welfare-maximizing CBDC nominal rate in the calibration is approximately 3.4%, beyond which welfare turns negative relative to the non-interest-bearing benchmark. These findings validate the theoretical prediction that while CBDC introduction can be welfare-improving, the optimal design entails a moderate rather than aggressive interest rate, with the precise level disciplined by the tradeoff between PM market efficiency and bank disintermediation.

1.1 Related literature

There has been a burgeoning number of CBDC papers recently that is impractical to review here, but my paper complements the CBDC papers in the New Monetarist literature. Keister and Sanches (2023) study the potential effects of introducing a CBDC on the banking system and monetary policy in perfectly competitive markets. In their model, banks face a pledgeability constraint. They find that a CBDC can lead to a disintermediation effect, where households substitute private bank liabilities for CBDC holdings, which can lead to a reduction in productive investment and social welfare. In contrast, my paper focuses on the conditions under which a CBDC could crowd out bank deposits and the policies required to implement the first-best allocation in an economy where CBDC and bank deposits coexist.

Chiu and Davoodalhosseini (2023) investigate the macroeconomic benefits of a cash-like and a deposit-like CBDC design. They show that a cash-like CBDC outperforms a deposit-like CBDC by raising consumption and welfare. In my paper, I explicitly incorporate the design choices around CBDC, such as the interest rate it pays and any associated fees, and studying how these policy parameters interact with household portfolio choices and banks' investment decisions. Household portfolio choices are also voluntary, meaning that there is no coercion or forced participation, so that sequential rationality is respected.

There are some influential papers that study the effect of CBDC issuance in economies with imperfect competition among banks. Andolfatto (2021) examines the impact of a CBDC on the banking system and monetary policy transmission when there is monopoly power in the banking system. He argues that a CBDC could discipline the banks by compelling them to increase their deposit rate, leading to an increase in bank deposits and financial inclusion. In Chiu et al. (2023), banks also have market power in the deposit market. They find that CBDC issuance could expand bank intermediation if the interest rate on CBDC lies within an intermediate range and causes disintermediation only if the interest rate is too high.

Williamson (2022b) focuses on efficiency where the central bank competes with the private sector for safe assets. Welfare is increased through households substituting CBDC for private bank liabilities and a CBDC may disintermediate banks when there is an overaccumulation of capital. This implies that disintermediation comes at the expense of improving economic efficiency. In my paper, the financial frictions in the banking sector themselves lead to an underproduction of goods but investment is too low to satisfy the demand. Lower investment in my model then reduces economic efficiency and bank deposits are priced at a premium.

Many studies have also investigated various aspects of CBDCs, such as their optimal design, their impact on monetary policy transmission, and their potential risks to financial stability.

Barrdear and Kumhof (2022) examine the macroeconomic effects of CBDC issuance in a DSGE model, while Davoodalhosseini (2022) examines the coexistence of cash and CBDC with balance-contingent transfers. Fernández-Villaverde et al. (2021) explore the effects of a CBDC on financial stability and bank runs within the framework established by Diamond and Dybvig (1983). Similar papers that also study CBDC and financial stability include Schilling et al. (2024), Keister and Monnet (2022), Rahman (2026), and Williamson (2022a). Brunnermeier and Niepelt (2019) and Niepelt (2024) show that a CBDC might not impact macroeconomic outcomes, including bank intermediation. Jiang and Zhu (2021) study the interactions between CBDC and reserves as tools for monetary policy. Various papers, including those by Agur et al. (2022), Davoodalhosseini and Rivadeneyra (2020), Wang (2023), and Kumhof and Noone (2018) contribute to understanding CBDC motivations and designs. However, none of these papers assume the absence of a government lump-sum transfer and consider individual rationality behind the interaction between CBDC and bank deposits. This is how my paper differs from this literature, and I also study the coexistence issues and which conditions are necessary for first-best allocations.

The remainder of the paper is organized as follows. Section 2 describes the physical environment. Section 3 characterizes agents' optimal decisions. Section 4 introduces the menu of policy instruments available to the government and the budget constraint that links them. Section 5 derives stationary monetary equilibria under alternative currency regimes. Section 6 studies optimal CBDC policies. Section 7 calibrates the model and assesses the quantitative effects of interest-bearing CBDC. Section 8 concludes.

2 The Physical Environment

The physical environment is based on Andolfatto et al. (2016) and Chiu and Davoodalhosseini (2023). Time is discrete and the horizon is infinite. As is typical in models like those in Lagos and Wright (2005), each period is divided into two subperiods. In this context, I refer to these subperiods as AM and PM, respectively. Search friction is abstracted away and agents meet in centralized locations in both subperiods. Two distinct perishable goods (or outputs) are produced and consumed in each subperiod called the AM good and the PM good.[5]

There is a continuum of infinitely-lived households, distributed uniformly on the unit interval $[0, 1]$. In each AM subperiod, a unit measure of new competitive bankers enters the economy, and they exit in the subsequent AM. Households are identical *ex ante*, but may differ *ex post*. Let $\{c_t(i), y_t(i)\} \in \mathbb{R}^2_+$ denote consumption and production of the PM good, respectively, at

[5]They can also be thought of *day good* and *night good*, respectively, as described in Andolfatto (2010). Provided that the two goods are unique and pertain to separate subperiods, the specific terminology used to label them is not critical to the analysis.

date t by agent i. Households discount utility payoffs across periods with the discount factor $0 < \beta < 1$; so that the preferences for household i are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U(x_t(i)) - Ah_t(i) + \pi \left[u(c_t(i)) - g(y_t(i)) \right] \right\}. \tag{1}$$

At the beginning of the PM, each member of a household experiences an idiosyncratic shock that determines their types. Let the types be classified as *consumers*, *producers*, and *idlers*.[6] A member of a household can become a consumer or a producer with equal probability π, so that the probability of becoming an idler is $1 - 2\pi$, where $0 < \pi \leq 1/2$. A consumer derives flow utility $u(c_t(i)) \in \mathbb{R}_+$ from consuming the PM good, where $u'' < 0 < u'$, and $u(0) = 0$, $u'(0) = \infty$. A producer derives flow utility $-g(y_t(i)) \in \mathbb{R}_+$ from producing the PM good, where $g(0) = g'(0) = 0$, $g' > 0$ for $y > 0$, and $g'' \geq 0$. The PM flow utility for idlers is normalized to zero. Since there is an equal measure of consumers and producers, feasibility and efficiency imply $c = y$.

In the AM subperiod, all households share identical preferences and opportunities. Their utility flow in the AM is given by $U(x_t(i)) - Ah_t(i)$, where $x_t(i) \in \mathbb{R}$ denotes the consumption of the AM good by individual i at date t, and $h_t(i)$ denotes their labor at date t. Assume that $U'' < 0 < U'$ with $U(0) = -\infty$ and $U'(0) = \infty$. The parameter A represents the relative emphasis households place on consumption versus labor in their utility preferences, a key factor that significantly influences the outcomes analyzed in this paper.

Bankers live for two periods, participate only in the AM, and consume only in old age. They are endowed with an investment technology. Households can consume the AM good, but they can also transfer these goods to young bankers. Young bankers can transform k units of the AM good into $f(k)$ units of the AM good in the next AM. The banker then consumes k in date $t + 1$ when he becomes old. The aggregate resource constraint in the AM is given by

$$X_t + k_{t+1} \leq H_t + f(k_t), \tag{2}$$

where $X \equiv \int x_t(i)di$ and $H \equiv \int h_t(i)di$.

As the PM good is perishable, another aggregate resource constraint requires

$$\int c_t(i)di \leq \int y_t(i)di \tag{3}$$

for all $t \geq 0$.

Consider a planner who weights all the agents equally and maximizes the *ex ante* utility of

[6]The idlers are inactive agents or nonparticipants who are intended to mimic the unmatched agents in Lagos and Wright (2005).

the agents with preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \{U(X_t) - AH_t + \pi [u(c_t(i)) - g(y_t(i))]\} \tag{4}$$

subject to the aggregate resource constraints (2) and (3). The steady-state first-best allocation constitutes a set of numbers (X^*, k^*, y^*) satisfying:

$$U'(X^*) = A, \tag{5}$$

$$\beta f'(k^*) = 1, \tag{6}$$

$$u'(y^*) = g'(y^*). \tag{7}$$

Lemma 1 is directly derived from the results presented in equations (5) through (7).

Lemma 1 *X^* is strictly decreasing in A. k^* and y^* are determined independently of A.*

3 Agent Decision-making

I will impose restrictions on the environment that will render trade by credit to become infeasible, so that a medium of exchange is essential. A medium of exchange is essential in the sense that it will allow society to achieve desirable outcomes that could not be achieved in its absence. Firstly, I assume limited commitment among household members, contrasting with banks' ability to commit and enforce debt repayment. Limited commitment implies that all trade is voluntary, respecting sequential rationality. This leads to the absence of a lump-sum tax instrument, a point I will discuss later.[7] Secondly, I assume household anonymity, which, combined with the first assumption, rule out private debt between households and makes a medium of exchange essential. Thirdly, I assume that households engage in a sequence of competitive spot market trades, exchanging bank deposits and interest-bearing CBDC for goods in both subperiods. In the following discussion, I explore how bank deposits and CBDC, which are voluntary payment instruments chosen by individual household members, possess distinct properties that are essential in determining the welfare consequences of monetary policy.

[7]This restriction follows Andolfatto (2010) and Andolfatto et al. (2016): the same anonymity and limited-commitment frictions that rule out private credit between households also prevent the government from imposing non-voluntary lump-sum transfers, since the government cannot identify or coerce individual agents any more effectively than private creditors can.

3.1 Banker decision-making

I first consider the decision-making of bankers who derive utility from consuming the AM good in old age. All markets are assumed to be competitive. Each of the bankers possesses an investment technology that allows them to invest in k units of the AM good at date t. The banker then produces $f(k)$ units of the AM good at date $t+1$. Assume $f'' < 0 < f'$, $f'(0) = \infty$, and $f'(\infty) = 0$. The banker finances the investment by issuing deposits d and pays a gross real interest rate R^D to each member of a household who is using bank deposits for payment. Figure 1 presents the timeline for all agents.

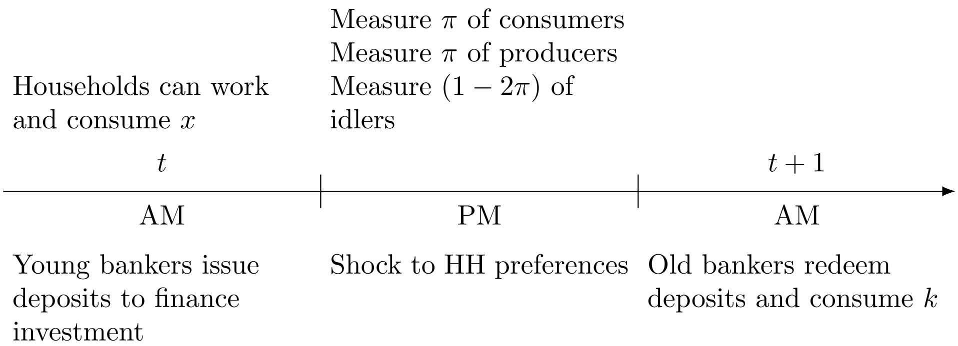

Figure 1: Timeline

Each banker takes the deposit price ϕ_1 in the AM as given and maximizes their profit

$$\max_k \quad \left\{\phi_1 f(k) - R^D k\right\}. \tag{8}$$

The first-order condition is then given by

$$f'(k) = \frac{R^D}{\phi_1}. \tag{9}$$

Given that f is an increasing and strictly concave function, condition (9) implies that the demand for investment k is decreasing in R^D.

Lemma 2 *The bankers' demand for investment spending $k(R^D)$ is decreasing in the deposit rate R^D.*

3.2 Household-member decision-making

Members of a household use CBDC and bank deposits as payment instruments. Denote by $\{(v_1, v_2), (\phi_1, \phi_2)\}$ the price of CBDC and bank deposits in the AM and PM markets, respectively.

Government policy pertinent to household decision-making will be described in detail below. Here, I outline key policy elements that impact the choices of individual household members. The government's policy rule operates before the start of the AM-market trading. A household member enters the AM with money balances in the form of CBDC and bank deposits, denoted by z and a respectively. The individual then has the option to approach either a government or a bank counter to transform these balances into $R^M z - \tau$ or $R^D a + f(a)$ units of money, respectively. Household members also have to pay a labor income tax $\tau_h \in [0, 1)$ on their labor income wh, where h is an individual's labor supply and w is the market price for leisure. If $R^M > 1$ and $\tau > 0$, then CBDC here is akin to an interest-bearing bond subject to a fixed fee, as in Andolfatto (2010).

If an individual member of a household decides not to use CBDC, he simply uses bank deposits. Subsequently he enters the AM-market with $R^D a + \phi_1 f(a)$ units of money in the form of bank deposits. In contrast to CBDC, bank deposits are partially illiquid financial instruments not issued by the government. The $f(a)$ component captures the illiquid aspect of bank deposits. I model this illiquidity aspect to capture the real-world diversity of bank deposits, differentiating between more liquid forms like checkable deposits and less liquid forms such as time deposits.[8]

After activities in the AM market, household members carry their CBDC and bank deposit balances into the PM market, where their roles as producers, consumers, or idlers are realized. Subsequently, following the PM market transactions, individuals retain their remaining CBDC and deposit balances, moving into the next AM. There, they are again faced with the choice between using CBDC or bank deposits, each offering different interest rates.

3.2.1 The AM market

Denote by $(z, a) \geq 0$ a household member's CBDC and bank deposit balance, respectively, in the AM at date t; and denote by $(m, d) \geq 0$ the CBDC and bank deposit balance, respectively, that the individual household member carries forward into the PM market. Let $\sigma \in [0, 1]$ denote the probability of an individual household member exercising the CBDC interest vehicle option. This σ also represents the probability of paying the fixed CBDC fee. Depending on the decision to pay the fee or not, the individual household member can then purchase or sell output x at

[8] For a more detailed exploration of the dynamics of various forms of exchange media, consider the insights offered in the studies by Chiu et al. (2023) and Wright (2010), which delve into the complexities and implications of different exchange mechanisms.

either the market price v_1 if selecting CBDC as currency, or the market price ϕ_1 if selecting bank deposits as the preferred currency of use for transactions. The AM-market budget constraint is then given by

$$x = (1 - \tau_h)\, wh + \sigma \left(R^M v_1 z - \tau \right) + (1 - \sigma)\phi_1 \left(R^D a + f(a) \right) - v_1 m - \phi_1 d. \tag{10}$$

A recursive representation of a household member's optimal choice problem is as follows. Let $W(z, a)$ denote the value function in the AM with CBDC and bank deposit balances, $(z, a) \geq 0$, respectively; and let $V(m, d)$ denote the value function in the PM before the household member realizes his type. The value functions $W(z, a)$ and $V(m, d)$ must satisfy the following recursive relationship:

$$\begin{aligned} W(z, a) \equiv \max_{x,h,\sigma,m,d} \quad & \{U(x) - Ah + V(m, d)\} \\ \text{s.t.} \quad & x = (1 - \tau_h)\, wh + \sigma \left(R^M v_1 z - \tau \right) \\ & + (1 - \sigma)\phi_1 \left(R^D a + f(a) \right) - v_1 m - \phi_1 d. \end{aligned} \tag{11}$$

Assuming that $V(m, d)$ is strictly concave, substituting out for h yields the following first-order conditions

$$U'(x) = \frac{A}{(1 - \tau_h)\, w}, \tag{12}$$

$$v_1 = \frac{(1 - \tau_h)\, w V_1(m, d)}{A}, \tag{13}$$

$$\phi_1 = \frac{(1 - \tau_h)\, w V_2(m, d)}{A}. \tag{14}$$

The demand for both CBDC and bank deposits, respectively, is independent of a household member's initial CBDC and deposit holdings, z and a, respectively. This implies that all household members enter the PM market with identical CBDC and deposit holdings. By comparing conditions (5) and (12), labor taxes introduce a wedge that raises $U'(x)$ above $U'(x^*)$; by the strict concavity of U, this implies underconsumption in the equilibrium relative to the optimum, that is, $x < x^*$. Moreover, the optimal CBDC interest vehicle choice must satisfy

$$\sigma = \begin{cases} 1 \\ [0, 1] \\ 0 \end{cases} \quad \text{if } R^M v_1 z - \tau \begin{cases} > \\ = \\ < \end{cases} \phi_1 \left(R^D a + f(a) \right), \tag{15}$$

so that the act of CBDC fee payment is sequentially rational if, and only if,

$$R^M v_1 z - \tau \geq \phi_1 \left(R^D a + f(a) \right). \tag{16}$$

For a given CBDC interest vehicle choice σ, by the envelope theorem:

$$W_1(z, a) = \frac{A\sigma R^M v_1}{(1 - \tau_h) w}, \tag{17}$$

$$W_2(z, a) = \frac{A(1 - \sigma)\phi_1 \left(R^D + f'(a)\right)\phi_1}{(1 - \tau_h) w}. \tag{18}$$

Given the assumptions that $R^M > 1$ and $\tau > 0$, the function $W(z, a)$ is characterized as piece-wise linear and convex in z and a. Furthermore, $W(z, a)$ is non-differentiable at the point $R^M v_1 z - \tau = \phi_1 \left(R^D a + f(a)\right)$.

3.2.2 The PM market

After AM-market activity, the household member carries CBDC and deposit balances with him into the PM market. Just before entering the PM market, the individual experiences a stochastic shock, where he realizes he is a consumer, a producer, or an idler. Following PM-market activity, the individual carries any remaining CBDC and deposit balances forward to the next AM, where he once again decides whether to exercise the CBDC interest vehicle option. Let $V^C(m, d)$, $V^P(m, d)$, and $V^I(m, d)$ denote the utility value associated with being a consumer, a producer, and an idler, respectively. The ex ante value function associated with entering the PM market is given by

$$V(m, d) = \pi V^C(m, d) + \pi V^P(m, d) + (1 - 2\pi) V^I(m, d). \tag{19}$$

A consumer who enters the PM with a wealth portfolio (m, d) faces the budget constraint $c = v_2(m - z^+) + \phi_2(d - a^+)$. Substituting out for c, the choice problem can be stated as

$$V^C(m, d) \equiv \max_{z^+ \geq 0, a^+ \geq 0} \left\{ u\left(v_2\left(m - z^+\right) + \phi_2\left(d - a^+\right)\right) + \beta W\left(z^+, a^+\right) \right\}. \tag{20}$$

In what follows, the consumer's debt-constraint $\{(z^+, a^+) \geq (0, 0)\}$ will play an important role in the results below. It is also important to note that if $z^+ = 0$, then $a^+ = 0$, and conversely. The implication of this assumption is that a consumer returning to the AM-market will likely find it optimal to refrain from exercising the CBDC interest vehicle option, meaning they will not pay the CBDC fee. By making use of (17) and (18), the PM consumption is characterized by

$$
\left.
\begin{aligned}
v_2 u'(c) &= \tfrac{\beta A R^M v_1^+}{(1-\tau_h)w} \\
\phi_2 u'(c) &= \tfrac{\beta A (R^D + f'(a^+))\phi_1^+}{(1-\tau_h)w}
\end{aligned}
\right\} \quad \text{if} \quad v_2 m + \phi_2 d \geq c
$$

$$
c = v_2 m + \phi_2 d \qquad \text{otherwise.} \tag{21}
$$

By the envelope theorem:

$$V_1^C(m,d) = v_2 u'(c), \tag{22}$$

$$V_2^C(m,d) = \phi_2 u'(c). \tag{23}$$

A producer who enters the PM with a wealth portfolio (m,d) faces the budget constraint $y = v_2\left(z^+ - m\right) + \phi_2\left(a^+ - d\right)$. Substituting out for y, the choice problem can be stated as

$$V^P(m,d) \equiv \max_{z^+\geq 0, a^+ \geq 0} \left\{-g\left(v_2\left(z^+ - m\right) + \phi_2\left(a^+ - d\right)\right) + \beta W\left(z^+, a^+\right)\right\}. \tag{24}$$

Since a producer has no desire to consume, his debt-constraint is necessarily slack. Therefore, a producer must strictly prefer to exercise his CBDC interest vehicle option, meaning he will pay the CBDC fee the next AM. Utilizing (17) and (18), the PM production is characterized by

$$v_2 g'(y) = \frac{\beta A R^M v_1^+}{(1-\tau_h)w}, \tag{25}$$

$$\phi_2 g'(y) = \frac{\beta A (R^D + f'(a^+))\phi_1^+}{(1-\tau_h)w}. \tag{26}$$

Idle household members entering the PM market with a wealth portfolio (m,d) simply carry their CBDC and bank deposit balances forward to the next AM. Consequently, we have $V^I(m,d) \equiv \beta W(m,d)$. The envelope theorem yields the following equations, applicable to both idlers and producers:

$$V_1^P(m,d) = V_1^I(m,d) = v_2 g'(y), \tag{27}$$

$$V_2^P(m,d) = V_2^I(m,d) = \phi_2 g'(y). \tag{28}$$

As the choice of a preferred payment instrument also comes into question, I want to restrict attention to equilibria where both bank deposits and CBDC coexist. For this to occur, the following rate-of-return equality condition must be satisfied:

$$\frac{R^M v_1^+}{v_2} = \frac{\left(R^D + f'(a^+)\right)\phi_1^+}{\phi_2}. \tag{29}$$

That is, for both assets to be accepted as payment, the expected rate of return on assets from the PM to the next AM must be the same. Consequently, at the individual level, portfolio composition becomes indeterminate in equilibrium.

4 Government Policy

I will now outline the government's policy. As a reminder, the government's operational approach involves intervening before AM-market trading begins. The policy entails offering a nominal interest rate of R^M on CBDC balances to household members willing to pay the fixed CBDC fee τ. Additionally, the government has the authority to impose labor income taxes, denoted as τ_h, on the labor earnings (wh) of individual household members, irrespective of their choice of currency.[9]

Let M^- denote the supply of outside money in the form of CBDC at the beginning of the AM-market (prior to any injection or withdrawal). Assume that this digital money supply grows at the constant (gross) rate $M = \mu M^-$, where M denotes the supply of digital money in the "next" period. Based on the assumptions, the initial CBDC supply M^- is entirely held by producers and idlers at the beginning of the AM. This is because both producers and idlers find it optimal to pay the CBDC fee τ. Consequently, the government bears an aggregate interest obligation of $(R^M - 1)M^-$, along with revenue from labor income tax $\tau_h wH$, and revenue from CBDC fee payments, $(1 - \pi)\tau$.

The government can also earn seigniorage revenue by printing new digital money $M - M^-$. Thus, a feasible government policy will have to satisfy the government budget constraint:

$$\underbrace{\left(R^M - 1\right) M^-}_{\text{Government spending}} = \underbrace{\tau_h wH}_{\text{Labor income tax revenue}} + \underbrace{(1-\pi)\,\tau}_{\text{CBDC fee revenue}} + \underbrace{M - M^-}_{\text{Seigniorage revenue}}. \tag{30}$$

By defining $\delta \equiv {}^{R^M}/_{\mu}$ and rearranging the equation above, the government budget constraint may alternatively be expressed as:

$$\tau = \frac{(\delta - 1)M - \tau_h wH}{1 - \pi}. \tag{31}$$

[9] In this context, the approach to implementing a labor tax differs significantly from that presented in Rahman and Wang (2026). In their paper, τ_h can also be viewed as a sales tax.

Invoking the results derived from the aforementioned assumptions, which are established to be valid within this class of quasilinear models, the joint equilibrium distribution of CBDC and bank deposit holdings (z, a) will be massed over points: $\{(0,0), (M, D), (2M, 2D)\}$. This means that the mass π of PM consumers enter the AM with zero units of CBDC and deposit holdings, the mass $(1-2\pi)$ of PM idlers enter with (M, D) units of wealth, and the mass π of PM producers enter with $(2M, 2D)$ units of wealth. Hence, an incentive-feasible government policy is one designed to ensure that both the fraction $(1-2\pi)$ of idlers and the fraction π of producers voluntarily pay the CBDC fee τ, while also satisfying (31) with the policy parameters (δ, τ, τ_h). It is worth noting that unlike the CBDC fee τ, the labor income tax τ_h will be voluntarily paid by all household members.

I define a passive policy as a government policy with constant money supply ($\mu = 1$) and the property $(\delta, \tau, \tau_h) = (1, 0, 0)$, which together imply $R^M = 1$. In a passive policy, CBDC is non-interest-bearing. Any policy that does not meet this criterion is referred to below as an *active policy*, where CBDC bears interest.

5 Stationary Monetary Equilibrium

In this section, I will examine the characteristics of stationary monetary equilibria under different currency regimes. These regimes include economies where only bank deposits serve as exchange media, economies where only CBDC is used, and economies where both bank deposits and CBDC coexist and compete as exchange media. My primary focus is on analyzing the properties of a stationary equilibrium where both CBDC and bank deposits coexist, given an incentive-feasible government policy. Briefly outlined, such equilibria must meet the following requirements: (i) Household and banker decisions are optimal; (ii) decisions are symmetric across all producers and consumers; (iii) markets clear at every date; and (iv) all real quantities remain constant over time.

5.1 A CBDC economy

Suppose that outside money, specifically CBDC, can only be used for payment in the PM. In particular, the rate of return on CBDC is higher than that on bank deposits. That is

$$\frac{R^M v_1^+}{v_2} > \frac{\left(R^D + f'(a^+)\right) \phi_1^+}{\phi_2}.$$

The market-clearing conditions for the money market are given by $m = M$ and $c = y$, as well as $v_2 = {}^{y}\!/_{M}$.

Gathering restrictions implied by individual behavior, I combine (13), (22), (27) to form

$$\frac{Av_1}{(1-\tau_h)w} = v_2 \left[\pi u'(c) + (1-\pi)g'(y)\right]. \tag{32}$$

Updating the latter expression by one period and combining with (25) yields

$$g'(y) = \beta R^M \left(\frac{v_2^+}{v_2}\right) \left[\pi u'(y^+) + (1-\pi)g'(y^+)\right]. \tag{33}$$

Restricting our attention to steady-state ($y = y^+ > 0$) it follows that ${}^{v_2^+}\!/_{v_2} = {}^{v_1^+}\!/_{v_1} = {}^{1}\!/_{\mu}$. This then together with the market-clearing conditions yields

$$\beta\delta L(y) = 1, \tag{34}$$

where

$$L(y) = \frac{\pi u'(y) + (1-\pi)g'(y)}{g'(y)}. \tag{35}$$

Market clearing implies

$$v_2 M \geq y^* \text{ or } v_2 M = y < y^*. \tag{36}$$

Conditions (16), (34) and (36) characterize the monetary equilibrium as a function of parameters, contingent upon an incentive-feasible policy $\delta \equiv {}^{R^M}\!/_{\mu}$ in an economy with CBDC as the only medium of exchange. Note that $L'(y)$ can either increase or decrease with respect to y, and $L(y^*) = 1$. Furthermore, it should be emphasized that the "standard" Friedman rule, where $(R^M, \mu) = (1, \beta)$ or $\delta = {}^{1}\!/_{\beta}$, is not incentive-feasible, as the CBDC fee is not voluntarily paid by every individual.

5.2 A bank credit economy

Now, let us consider the case where inside money, specifically bank deposits, can only be used for payment in the PM. Conversely, the condition below holds:

$$\frac{R^M v_1^+}{v_2} < \frac{\left(R^D + f'(a^+)\right)\phi_1^+}{\phi_2}.$$

This implies that the return on bank deposits is higher than that on CBDC. The market-clearing conditions for the deposit market are given by $d = D$ and $c = y$.

Gathering restrictions implied by individual behavior, I combine (14), (23), (28) to form

$$\frac{Av_1}{(1-\tau_h)w} = \phi_2 \left[\pi u'(c) + (1-\pi)g'(y)\right]. \tag{37}$$

Updating the latter expression by one period and combining with (26) yields

$$g'(y) = \beta\left(R^D + f'(a^+)\right)\left(\frac{\phi_2^+}{\phi_2}\right)\left[\pi u'(y^+) + (1-\pi)g'(y^+)\right]. \tag{38}$$

Restricting our attention to steady-state ($y = y^+ > 0$, $a = a^+ = 0$) it follows that $\phi_2^+ = \phi_2 > 0$ and $\phi_1^+ = \phi_1 > 0$. This then together with the market-clearing conditions yields

$$\beta\left(R^D + f'(a)\right)L(y) = 1. \tag{39}$$

To solve for the AM price of bank deposits assume that $L(y^*) = 1$, so that there is zero-liquidity premium at the first-best allocation and that assets are efficiently priced at their "fundamental level". By combining the banker's first-order condition (9) with (39), we obtain:

$$\phi_1^* = \frac{\beta R^D}{1-\beta R^D} > 0, \tag{40}$$

which bears resemblance to the standard asset-pricing formula derived for risk-neutral agents. Note that ϕ_1 is increasing in the bank interest rate R^D. Furthermore, we need $1 < R^D < 1/\beta$ to satisfy $0 < \phi_1 < \infty$ for a bank credit equilibrium to exist.

To solve for the PM price of bank deposits, we can combine (26) and (40) to find:

$$\phi_2^* = \frac{\beta A R^D}{(1-\beta R^D)(1-\tau_h)wg'(y^*)} > 0. \tag{41}$$

An immediate observation from the above equation is the influence of labor income tax τ_h on deposit prices. Once again, market clearing implies

$$\phi_2^* D \geq y^* \text{ or } \phi_2 D = y < y^*. \tag{42}$$

Conditions (9), (39), (40), (41), and (42) constitute the key restrictions that characterize the general equilibrium allocation and price system in this competitive economy, where only bank deposits are used as the medium of exchange. If the debt-constraints are binding, bank deposits may become overvalued ($\phi_1 > \phi_1^* \implies \phi_2 > \phi_2^*$) due to a shortage in their supply. Consequently, this creates a liquidity premium on the price of bank deposits, leading to a lower expected rate of return. The following condition must hold for deposit prices to be overvalued with binding debt-constraints:

$$\frac{1}{\beta} > \frac{R^D(1+\phi_1)}{\phi_1} > 1.$$

5.3 A mixed CBDC and bank credit economy

I now consider an economy where CBDC and bank deposits coexist as payment instruments, with condition (29) being satisfied. Market clearing now implies

$$v_2 M + \phi_2^* D \geq y^* \text{ or } v_2 M + \phi_2 D = y < y^*. \tag{43}$$

To confirm whether the conjecture made regarding (43) holds in equilibrium, we can use (21) to derive:

$$A \geq \frac{(1-\tau_h) w g'(y^*) y^*}{\beta \left[\delta v_1 M + (R^D + f'(a))\phi_1^* D\right]}. \tag{44}$$

Since we can use (40) to derive an equilibrium value for $\phi_1 > 0$, then all we require is any value $v_1 < \infty$ satisfying (44) to obtain an equilibrium in which both CBDC and bank deposits coexist. In economies with a sufficient level of A, multiple assets can coexist and hold value.

To derive the consumption allocation across household types in each AM, we can combine the joint steady-state distribution of wealth above with the household budget constraint (10). For those who were consumers in the previous PM, $\sigma = 0$ and $(z^+, a^+) = (z, a) = (0, 0)$, so:

$$x = (1-\tau_h)\, wh - v_1 M - \phi_1 D. \tag{45}$$

For those who were idlers in the previous PM, $\sigma = 1$ and $(z^+, a^+) = (z, a) = (M, D)$, so:

$$x = (1 - \tau_h)\, wh + (R^M - 1)v_1 M - \tau - \phi_1 D. \tag{46}$$

For those who were producers in the previous PM, $\sigma = 1$ and $(z^+, a^+) = (z, a) = (2M, 2D)$, so:

$$x = (1 - \tau_h)\, wh + (2R^M - 1)v_1 M - \tau - \phi_1 D. \tag{47}$$

We can easily verify that the population-weighted sum of (45), (46), and (47) is nonzero.[10]

Recall that we want to restrict our attention to incentive schemes that satisfy (16), ensuring that producers strictly prefer to pay the CBDC fee, while idle household members weakly prefer to do so. Consider that idlers will find it optimal to pay the CBDC fee the next AM. In equilibrium, idlers enter the AM market with wealth $(z, a) = (M, D)$. By appealing to (31), the CBDC fee constraint $R^M v_1 z - \tau \geq \phi_1 \left(R^D a + f(a)\right)$ can be written as follows:

$$(1 - \pi)\left[R^M v_1 M - \phi_1(R^D D + f(D))\right] + \tau_h w H \geq (\delta - 1)M. \tag{48}$$

In this class of models, it is important to consider the sequential participation of a consumer who enters the AM market with zero CBDC and deposit balances. To accumulate wealth, this agent must exert significant effort, sacrificing transferable utility. Let $W(0, 0)$ denote the payoff for rebalancing asset holdings in the AM on the equilibrium path. If the cost or sacrifice of rebalancing is excessively high, an individual household member would forego that opportunity and enter the next PM with zero CBDC and deposit holdings. However, the individual can still consume in the AM and opt to work in the PM market. If he chooses to work in the PM, then he takes his CBDC and deposit holdings and spend them in the AM. Along this alternative path, the individual never consumes in the PM. Let $\widehat{W}(0, 0)$ represent the payoff of this alternate strategy.[11] Then, sequential rationality must satisfy:

$$W(0, 0) \geq \widehat{W}(0, 0). \tag{49}$$

Another constraint that must be met is that producers in the PM market must be willing to produce good y for (z, a) units of money. It is straightforward to verify that this constraint is satisfied in equilibrium.

Condition (43) and (49), along with conditions (9), (16), (34), (39), (40), and (41) derived earlier, constitute the key restrictions characterizing the general equilibrium allocation and price

[10] Although the process may appear similar, the result differs significantly from that of Andolfatto (2010).

[11] A formal derivation of $\widehat{W}(0, 0)$ is provided in the Appendix as part of the proof of Lemma 4.

system in an economy where both CBDC and bank deposits hold value.

In the next section, I will examine the various policies required for first-best implementation in a mixed CBDC and credit economy.

6 Optimal Policies

I now study the implementation of first-best allocation in a mixed CBDC and bank credit economy. Depending on parameters, the level of output may or may not be efficient. The goal is to identify the policies that are required for a first-best implementation.

6.1 Passive policy

I first determine the conditions under which a passive policy $(\delta, \tau, \tau_h) = (1, 0, 0)$ is optimal. With a passive policy where CBDC is non-interest bearing, $\sigma = 1$ holds trivially for all household members. Hence, the CBDC fee constraint (48) can be ignored. The government budget constraint (31) will also be satisfied, so that the passive policy is trivially an incentive-feasible policy. The question is whether the consumer's debt-constraint (21) will bind or not. From (44), if the consumer debt-constraint is slack then the conjecture that needs to be satisfied is the following:

$$A \geq \frac{wg'(y^*)y^*}{\beta \left[v_1 M + (R^D + f'(a))\phi_1^* D\right]}. \tag{50}$$

Proposition 1 *Under a passive government policy* $(\delta, \tau, \tau_h) = (1, 0, 0)$ *with non-interest bearing CBDC, there exists a unique* $0 < A_0 < \infty$ *that satisfies* $v_2 M + \phi_2 D = y^*$.

For economies with $A \geq A_0$*, the competitive monetary equilibrium is efficient. That is,* (40) *and* (41) *hold, as well as*

$$v_2 M + \phi_2 D \geq y^*. \tag{51}$$

For economies with $0 < A < A_0$*, the competitive monetary equilibrium is inefficient. That is,*

$$v_2 M + \phi_2 D < y^*, \tag{52}$$

$$\phi_1 > \phi_1^*, \tag{53}$$

$$\phi_2 > \phi_2^*. \tag{54}$$

According to Proposition 1, if $A \geq A_0$, the household's preference for the AM good is sufficiently high that the consumer debt-constraint remains slack, even though the CBDC is non-interest-bearing. CBDC and bank deposits are priced at their fundamental level, with $\phi_1 = \phi_1^*$ and $\phi_2 = \phi_2^*$ as in (40) and (41). The deposit rate R^D settles in the interval $(1, 1/\beta)$ required for the asset-pricing condition to admit a finite, positive ϕ_1^*, and producers willingly deliver the first-best level of PM output $y = y^*$, where by Lemma 1, y^* is determined independently of A. The AM allocation is governed by the household first-order condition (12) with $\tau_h = 0$, which leads to $U'(X) = A/w$; comparing with the planner's condition (5) and using concavity of U, the equilibrium AM allocation satisfies $X = X^*$ if $w = 1$, $X < X^*$ if $w < 1$, and $X > X^*$ if $w > 1$. The same wage-dependent wedge between X and X^* applies at every level of A.

On the other hand, if $A < A_0$, the consumer debt-constraint binds and the inefficiency at the PM and asset-price margins becomes substantive. The shortage of liquidity is reflected in a liquidity premium on bank deposits, with $\phi_1 > \phi_1^*$ and $\phi_2 > \phi_2^*$. To clear the deposit market under this shortage, R^D rises above its frictionless level; by Lemma 2, the higher R^D depresses bankers' investment demand, so $k < k^*$, where by Lemma 1 the first-best k^* is also determined independently of A. The binding debt-constraint then forces PM consumption below the first-best, $y < y^*$. The AM allocation is still governed by (12), so the wage-dependent wedge $X - X^*$ described above is unchanged by crossing the A_0 threshold.

Proposition 1 is related to the nonmonetary and monetary equilibria discussed in Lagos and Rocheteau (2008) and Andolfatto et al. (2016). Lagos and Rocheteau (2008) find that if the capital stock is small, there is an overaccumulation of capital in equilibrium, leading to lower consumption in the PM. Conversely, Andolfatto et al. (2016) assume an efficient capital stock, resulting in an overvaluation of money (backed by the capital stock) in equilibrium and lower PM consumption. The mechanism in my model differs from both. As in Lagos and Rocheteau (2008), the inefficiency operates through the capital-investment channel, but with the opposite sign: the binding consumer debt-constraint at $A < A_0$ produces *under*-investment ($k < k^*$) rather than over-accumulation, because the elevated R^D depresses bankers' demand for AM goods (Lemma 2). As in Andolfatto et al. (2016), the shortage of liquid assets is capitalized into an overvaluation of the payment instrument, but here the premium accrues to bank deposits ($\phi_1 > \phi_1^*$, $\phi_2 > \phi_2^*$) rather than to outside money. The resulting liquidity premium on deposits is closest in form to that of Keister and Sanches (2023), except that their result depends on β rather than A.

When $A < A_0$, policies need to be necessarily inflationary, that is, $\delta > 1$ ($R^M > 1$, $\mu > 1$), to restore efficiency. Moreover, the private banks will have to lower R^D so as not to reduce their investment demand.

6.2 Active policies

If first-best implementation under a passive policy is infeasible for economies with low A, then our only recourse is a range of active policies necessitating strictly positive inflation and nominal interest rates. This implies an interest-bearing CBDC is necessary to improve welfare. I now focus solely on economies where $A < A_0$, specifically in regions of the parameter space where a passive policy fails to implement the first-best allocation. This is because the real rate of return is too low to motivate bankers to issue deposits and producers to supply the first-best level of output. The government has three instruments to finance its CBDC interest obligation: seigniorage, labor income tax revenue and CBDC fee revenue. Below, I restrict attention to policies that satisfy $1 < \delta < {}^1\!/_\beta$; since otherwise, a monetary equilibrium will fail to exist, except in the limiting case of $\delta \nearrow \beta^{-1}$.

Case 1: $1 < \delta < {}^1\!/_\beta$, $\tau = 0$, *and* $\tau_h > 0$. In what follows, I ask whether seigniorage and labor income tax revenue help to implement the first-best allocation in the absence of CBDC fees. Note that from (44) it is easy to verify that A is decreasing in both δ and τ_h. As $A < A_0$, we will have

$$A_0 \equiv \frac{(1-\tau_h)wg'(y_0)y_0}{\beta\left[\delta v_1 M + (R^D + f'(a))\phi_1 D\right]}. \tag{55}$$

We simply need to reduce the right-hand side of condition (55) to relax the debt-constraint for consumers, ensuring that (44) holds. This can be accomplished through a strictly inflationary policy and a strictly positive labor income tax. Since A is decreasing in both δ and τ_h, there exists a unique critical value $0 < A_1 < A_0$ so that the consumer debt-constraint is slack. Note that since the labor income tax is distortionary (see condition (12)), AM consumption in the equilibrium is lower than in the optima. Unlike Andolfatto et al. (2016) and Andolfatto (2010), money injected in this manner is not superneutral when it is introduced in the form of interest. This is because seigniorage and labor income tax expand the set of economies that can attain the first-best allocation.

Case 2: $1 < \delta < {}^1\!/_\beta$, $\tau > 0$, *and* $\tau_h = 0$. In the absence of labor income tax with $\tau_h = 0$, I want to now show how CBDC expenditures can be financed by seigniorage and CBDC fees to improve the allocation. For this instance, we have

$$A_0 \equiv \frac{wg'(y_0)y_0}{\beta\left[\delta v_1 M + (R^D + f'(a))\phi_1 D\right]}. \tag{56}$$

The following lemma characterizes the optimal CBDC fee necessary to implement the first-best allocation.

Lemma 3 *For economies with $A_1 \leq A < A_0$, the optimal CBDC fee τ that can be attained to implement the first-best allocation is*

$$\tau^*(\delta, R^D, A) = (1-\pi)^{-1} \left\{ (\delta - 1)M + \frac{\beta AH\left[\delta v_1 M(1-\beta R^D) + R^D D\right]}{(1-\beta R^D)g'(y^*)y^*} - wH \right\}. \qquad (57)$$

$\tau^*(\delta, R^D, A)$ is increasing in A. $\tau^*(\delta, R^D, A)$ is increasing in δ and the effect of R^D on $\tau^*(\delta, R^D, A)$ is ambiguous.

Proof. See Appendix ■

Unlike in Andolfatto et al. (2016), the CBDC fee $\tau(\delta, R^D, A)$ here is increasing in A. Although the interpretation of A here and in their paper is similar, A enters slightly differently in my model. The CBDC fee is also not independent of inflation μ and interest rates R^M and R^D. As δ (the ratio of nominal interest on CBDC to money growth rate) increases, the CBDC fee also increases.

The positive effect of R^M on τ^* suggests that when the government pays a higher interest rate on CBDC, it can afford to charge a higher CBDC fee without significantly reducing the demand for CBDC. This is because the higher interest rate on CBDC compensates households for higher CBDC fees, maintaining the overall attractiveness of CBDC as a payment instrument. The effect of R^D on τ^* can potentially induce both income and substitution effects. If R^D increases, bank deposits become relatively more attractive compared to CBDC, prompting the government to potentially reduce CBDC fees to incentivize households to use CBDC instead of bank deposits (substitution effect). Conversely, as households earn more interest income from their bank deposits, they may have additional disposable income to spend on transaction fees. Consequently, the government could potentially charge a higher CBDC fee without significantly discouraging CBDC use (income effect). Ultimately, the sign of $\partial\tau^*/\partial R^D$ will depend on which effect dominates. However, for $A \geq A_0$, no CBDC fee income and inflation are necessary to implement the first-best allocation.

Since coercion is ruled out and all trade must be voluntary, it is necessary to determine the conditions under which the CBDC fee τ^* can be collected through voluntary contributions. Specifically, the CBDC fee τ^* must satisfy the CBDC fee constraint given by equation (16). This constraint ensures that the fee is set at a level that incentivizes agents to participate in the CBDC system voluntarily, without the need for coercion or forced participation.

It is also important to consider how CBDC could potentially displace bank deposits in an environment with strictly positive inflation and positive nominal interest rates. If $R^M > 1$, maintaining the rate-of-return equality condition (29) requires an increase in R^D. However, as Lemma 2 suggests, an increase in R^D would lead to a decline in investment demand. This is a channel through which bank deposits could be crowded out by CBDC, even though sequentialy rationality is respected.

Case 3: $1 < \delta < 1/\beta$, $\tau > 0$, *and* $\tau_h > 0$. Suppose now that $A < A_1$, indicating that the government cannot implement the first-best allocation exclusively with either labor income tax (and seigniorage) as in *Case 1*, or with a fixed fee (and seigniorage) as in *Case 2*. With further restrictions in the economy, the government will have to utilize all the available policy instruments at its disposal. This comes at the expense of giving up more degrees of freedom than desired.

Note that with the both $\tau_h > 0$ and $\tau > 0$, the CBDC fee constraint (48) needs to be satisfied to induce voluntary participation. Rearranging (48) further we can obtain the expression

$$\tau_h \geq \frac{(1-\pi)(R^D + f(D)) - [(1-\pi)R^M v_1 + 1 - \delta]M}{wH}. \tag{58}$$

Condition (58) is an expression for the minimum labor income when the government uses all the policy tools available in a constrained equilibrium. A higher CBDC fee and labor income tax relaxes the CBDC fee constraint, which is a channel through which households can relax their debt-constraint. Since $A < A_1$ and A decreases with both δ and τ_h, there exists a unique critical value $0 < A_2 < A_1 < A_0$ that will relax the consumer debt-constraint. The following proposition identifies the regions of the parameter space in which active government policies alone, within a constrained CBDC and credit equilibrium, are sufficient for achieving the first-best allocation.

Proposition 2 *If* $A_0 > A \geq A_1$*, the first-best allocation can be implemented with* $1 < \delta < 1/\beta$*,* $\tau = 0$*, and* $\tau_h > 0$*, or with* $1 < \delta < 1/\beta$*,* $\tau > 0$*, and* $\tau_h = 0$ *where CBDC is interest-bearing. If* $A_1 > A \geq A_2$*, implementing the first-best allocation requires* $1 < \delta < 1/\beta$*,* $\tau > 0$*, and* $\tau_h > 0$*. Price stability is achieved with* $\phi_1 = \phi_1^*$ *and* $\phi_2 = \phi_2^*$ *satisfying* (40)*,* (41) *and* (50)*.*

Propositions 1 and 2 identify three regions of the parameter space with different rates of returns on CBDCs. For $A \geq A_0$, a passive policy $((\delta, \tau, \tau_h) = (1, 0, 0))$ is sufficient to deliver a first-best allocation where CBDC bears zero interest. There is adequate liquidity provisioning in the economy, so having a constant supply of CBDC is enough to induce household members to work hard and produce the PM good at the first-best level. Bankers will issue deposits and pay interest rate R^D high enough so that household members sacrifice enough of their consumption to produce the AM good. Moreover, all trade is voluntary among agents as sequential rationality is respected. CBDC in this case is non-interest bearing with $R^M = 1$ and can be viewed as digital cash with which agents cannot hide their money balances. The unbacked nature of this form of government-issued fiat money over private money (bank deposits) is an advantage. There is no disintermediation in the banking system in this case as the rate-of-return equality condition (29) holds.

For $A_0 > A \geq A_1$, the constrained-efficient policy entails positive inflation and non-zero nominal interest rates, as a constant supply of CBDC is insufficient to restore efficiency. In this case, the government will require a combination of seigniorage with labor income tax ($1 < \delta < 1/\beta$, $\tau = 0$, and $\tau_h > 0$) or a combination of seigniorage with fee income ($1 < \delta < 1/\beta$, $\tau > 0$, and $\tau_h = 0$) to induce producers to produce an efficient level of PM goods and for bankers to issue enough deposits. CBDC in this case must be interest-bearing to deliver efficiency. However, CBDC could crowd out bank deposits if R^M is too high relative to R^D and decrease investment. For $A_1 > A \geq A_2$, the constrained-efficient policy requires the government to use both labor income tax and fee income (and seigniorage) to induce producers to produce y^* and for bankers to issue deposits k^*. Any optimal policy in this region must have a CBDC that bears interest. In the equilibrium, however, there will be an underproduction of the AM good X^* due to the distortionary effects of labor income tax. The optimal CBDC fee income and labor income tax through voluntary contributions are incentive-feasible. Strictly positive inflation and nominal interest rates encourage households to use CBDC and pay its associated fees, as it relaxes their CBDC fee constraint so that their debt-constraint does not bind. Disintermediation of banks could still occur with this policy as it is strictly inflationary with strictly positive interest rates.

Finally, it is worth pointing out that achieving the first-best allocation is impossible under deflation within any parameter space, in contrast to Andolfatto et al. (2016). The government policy here mirrors that of Andolfatto (2010), where an incentive-feasible policy precludes deflation. Unlike in Andolfatto et al. (2016), there is no dividend fee income from holding assets to encourage voluntary contributions for a small CBDC fee. Additionally, in Andolfatto et al. (2016), there are no competing payment instruments; however, in this model, both CBDC and bank deposits compete as exchange media, and their choice must adhere to individual rationality. Thus, positive inflation and non-zero nominal interest rates are always necessary in more impatient economies when CBDC and bank deposits compete as exchange media, as deflation is infeasible.

7 Quantitative Analysis

In this section, I calibrate the theoretical model to the US economy in order to assess the impact of introducing different versions of a CBDC. As noted earlier, the CBDC in my paper can either be non-interest-bearing or interest-bearing.

7.1 Calibration

For the quantitative exercise, I consider an annual model with utility functions $U(x) = B\log(x+\varepsilon)$ in the AM and $u(c) = ((c+\varepsilon)^{1-\chi} - 1)/(1-\chi)$ in the PM, PM disutility $g(y) = y^2/2$, and production technology $f(k) = k^{1-\theta}/(1-\theta)$, where ε is a utility normalization parameter set to 0.001. The quadratic PM disutility specification follows the standard convention in the New-Monetarist money-demand calibration literature, including Andolfatto et al. (2016) (whose framework I build on) and Aruoba et al. (2011). The production curvature $\theta = 0.397$ is set externally following Chiu and Davoodalhosseini (2023).

7.1.1 Derivation of the money demand curve

Before describing the calibration procedure, I derive the model-implied M1/GDP relationship that will be matched to the data. The derivation operates in the stationary mixed equilibrium where CBDC and bank deposits coexist as competing payment instruments and the rate-of-return equality condition (29) holds.

With $u'(y) = y^{-\chi}$ and $g'(y) = y$ from the functional-form assumptions above,

$$L(y) = \pi\, y^{-(\chi+1)} + (1-\pi).$$

The data target is the 3-month T-bill rate, while the model is parameterized in terms of δ. To connect them, I identify the empirical nominal interest rate with the model's internal liquidity premium: the function $L(y)$ measures the expected marginal valuation of money in the PM relative to the marginal disutility of producing the good required to acquire it, and the net premium $L(y) - 1$ is the excess real return that liquid balances provide. This liquidity premium is the unique object in the model that carries the interpretation of an opportunity cost of holding money. This identification is standard in the New Monetarist money-demand calibration literature; see, for example, Lagos and Wright (2005), Aruoba et al. (2011), and Chiu et al. (2023). I therefore identify

$$1 + i \;\equiv\; L(y) \;=\; \frac{1}{\beta\,\delta}. \tag{59}$$

At the Friedman ratio $\delta = 1/\beta$ the premium collapses to unity and $i = 0$; as δ falls, the premium widens and i rises. This matches the textbook interpretation of the short rate as the opportunity cost of holding liquid balances. The mapping also delivers a Fisher-style relation among the model's monetary primitives: combining (59) with $\delta = R^M/\mu$ gives $1+i = \mu/(\beta R^M)$, with $1/\beta$ playing the role of the gross real interest rate and μ/R^M playing the role of the effective gross inflation rate from the perspective of a money holder.

Substituting (59) into the expression for $L(y)$ yields the closed-form expression

$$y(i) = \left(\frac{\pi}{\pi + i}\right)^{1/(\chi+1)}. \tag{60}$$

PM output is strictly decreasing in i, with $y(0) = 1 = y^*$ at the Friedman rule and y falling monotonically as the nominal rate rises.

Model M1 is computed from market clearing in the binding-constraint regime. The aggregate value of liquidity in the PM equals total PM consumption: $\text{M1} = v_2 M + \phi_2 D = y$, so model M1 equals PM output $y(i)$ directly. Model GDP is composed of AM consumption, PM consumption (with mass π), and the bankers' investment return: $\text{GDP} = X + \pi y + f(k)$, where X and $f(k)$ are evaluated at their equilibrium values. The household AM optimality condition (12) at the wage normalization $w = 1$ gives $X = B(1 - \tau_h)/A$, and the bankers' first-order condition (9) at first-best gives $k^* = \beta^{1/\theta}$, so $f(k^*) = \beta^{(1-\theta)/\theta}/(1 - \theta)$.

Combining these, the model-implied money demand curve is

$$\frac{\text{M1}}{\text{GDP}}(i) = \frac{y(i)}{\dfrac{B(1-\tau_h)}{A} + \pi\, y(i) + \dfrac{\beta^{(1-\theta)/\theta}}{1-\theta}}, \tag{61}$$

with $y(i)$ given by (60). Equation (61) is the object compared against the empirical M1/GDP ratio at observed values of the T-bill rate.

7.1.2 Identification

Equation (61) makes the identification structure transparent. The slope of M1/GDP in i runs entirely through $y(i)$, which depends only on χ given the functional-form assumptions; the slope of the money-demand curve thus identifies χ. The level of M1/GDP at any fixed i depends on A, B, and τ_h only through the composite ratio $B(1 - \tau_h)/A$, since these parameters enter only via the AM consumption term in the denominator.

In the policy experiments, τ_h enters the household budget constraint and the labor-supply wedge as a substantive fiscal instrument distinct from the AM utility weight B. I therefore fix $\tau_h = 0.24$ from Christiano et al. (2014), set $A = 1$ as a productivity normalization, and let B absorb whatever residual the level moment requires.

Two further observations on identification. First, neither R^M nor μ enters the fitting loop directly. By the liquidity-premium identification (59), the implied policy ratio $\delta(i) = 1/[\beta(1+i)]$ is pinned at each observation by the data nominal rate i. The data identify only the ratio μ/R^M at the sample-average rate, not μ and R^M separately. I set $\mu = 1.02$ to match the historical long-term US inflation rate; R^M acquires substantive content as the policy variable, where it is varied across counterfactual experiments. Second, the production curvature θ enters only the constant term $f(k^*)$ in the denominator of (61); changing θ shifts the level of M1/GDP but not its slope, so θ is not separately identified from B within money demand.

7.1.3 Calibration procedure

The model is fit to the M1/GDP curve via a nested nonlinear least-squares procedure that follows Chiu et al. (2023). The procedure treats the preference shock probability π as a calibration target, while imposing the model restriction $0 < \pi \leq 1/2$.

The outer loop searches over $\pi \in [0.15, 0.40]$.[12] Values of π below 0.10 are excluded because they imply PM consumption opportunities occur less than once per decade at the annual frequency, which strains the model's interpretation of PM trade as recurring consumption events. For each candidate π in the outer grid, the inner loop solves

$$\min_{(\chi, B)} \sqrt{\frac{1}{N} \sum_{t=1}^{N} \left[\frac{\text{M1}}{\text{GDP}}(i_t; \chi, B, \pi) - \widehat{\text{M1/GDP}}_t \right]^2},$$

with parameter bounds $\chi \in [0.01, 3]$ and $B \in [0, 10]$. The outer rule selects the π that delivers the lowest inner-loop root mean squared error (RMSE).

I use the new M1 series from Lucas and Nicolini (2015), excluding the post-financial-crisis period when M1 demand rose sharply due to non-transactional motives. The choice of money aggregate in the data is M1 plus liquid deposits in the money market. Table 1 summarizes all the parameter values along with their calibration targets.

Parameters	Notation	Value	Calibration Targets
Calibrated externally			
Discount factor	β	0.97	Standard in literature
Money growth rate	μ	1.02	2% inflation
Labor income tax	τ_h	0.24	Christiano et al. (2014)
Production curvature	θ	0.397	Set directly
AM preference	A	1.00	Set directly
Calibrated internally			
Preference shock	π	0.150	Money demand curve
PM utility curvature	χ	0.162	Money demand curve
AM utility parameter	B	2.35	Money demand curve

Table 1: Calibration Results

[12] This range is anchored on the analogous DM-trading probability $\Omega = 0.22$ calibrated by Chiu et al. (2023) for the same data sample (1987–2008) using an equivalent nested NLS procedure, with reasonable slack to either side.

Figure 2 shows the predicted money demand curve from the model compared to the actual data from 1987 to 2008. The model captures the downward-sloping relationship between the M1/GDP ratio and the 3-month T-bill rate with a RMSE of 0.0159. The average M1/GDP ratio in the data is 0.232, and the model predicts an average of 0.232 as well.

Using these calibrated parameters, I now analyze the quantitative effects of introducing

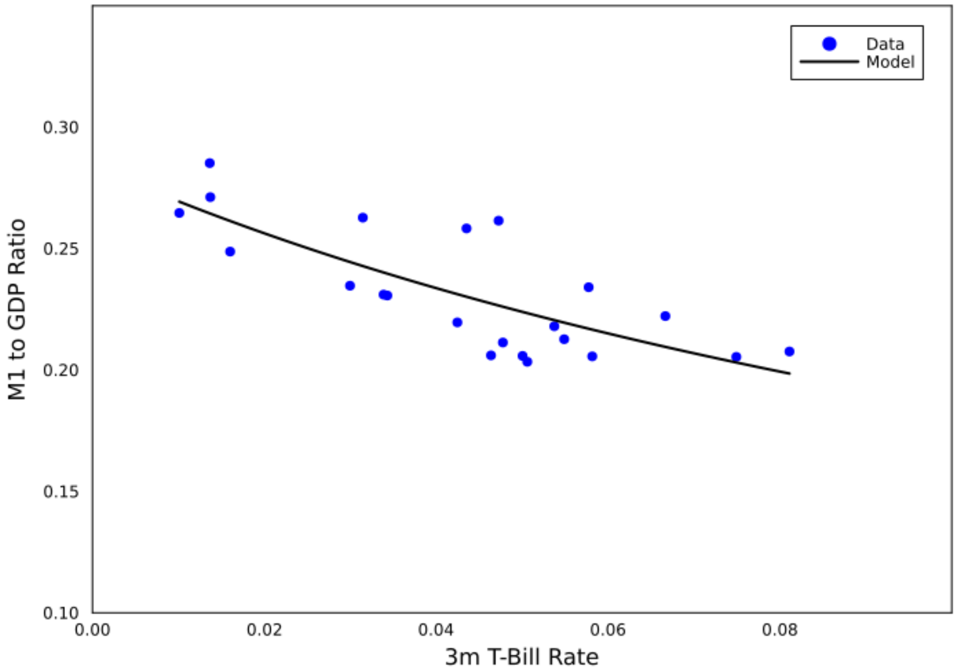

Figure 2: Money Demand Curve: Model vs Data

different versions of CBDC policy.

7.2 Effects of an interest-bearing and non-interest-bearing CBDC

The calibrated economy has $A = 1$ and lies in the impatient region $A < A_0$, so the relevant benchmark is Proposition 2, which implements the full first-best by jointly choosing (δ, τ, τ_h) with the optimal CBDC fee pinned by Lemma 3. The experiments below do not attempt that joint optimization; instead, they vary only the CBDC nominal rate R^M, holding μ at the 2% calibration target, τ_h at the Christiano et al. (2014) value of 0.24, and $\tau = 0$. This restriction reflects that μ and τ_h are pre-existing fiscal primitives, not central-bank levers, and that the relevant counterfactual is the marginal effect of introducing interest-bearing CBDC into an existing fiscal environment. The welfare optima reported below are therefore constrained or second-best.

Taking the economy with non-interest-bearing CBDC ($R^M = 1$, equivalently $\delta < 1$) as my benchmark, I trace out the effects of raising R^M toward the admissible upper bound, beyond which $\delta = R^M/\mu$ would exceed $1/\beta$ and monetary equilibrium would fail to exist. Within my calibration this admissible range is $R^M \in [1, 1.05]$, spanning a CBDC nominal interest rate from 0 to 5 percent. Figures 3 and 4 show the results for varying R^M and the equivalent policy ratio δ, respectively. Table 2 summarizes representative values.

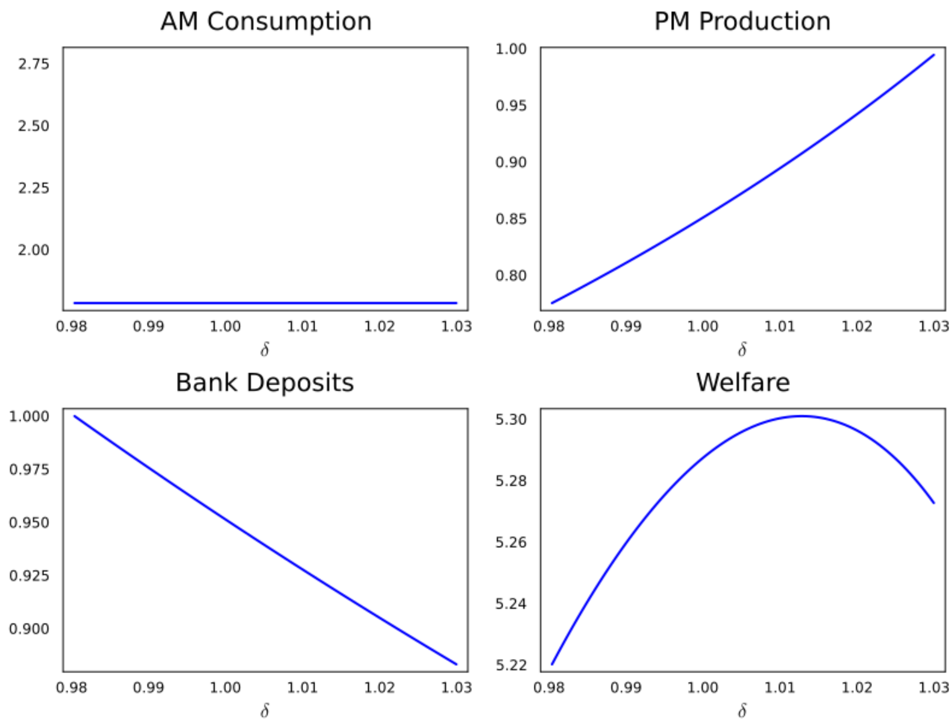

Figure 3: Effects of Interest-bearing CBDC through δ

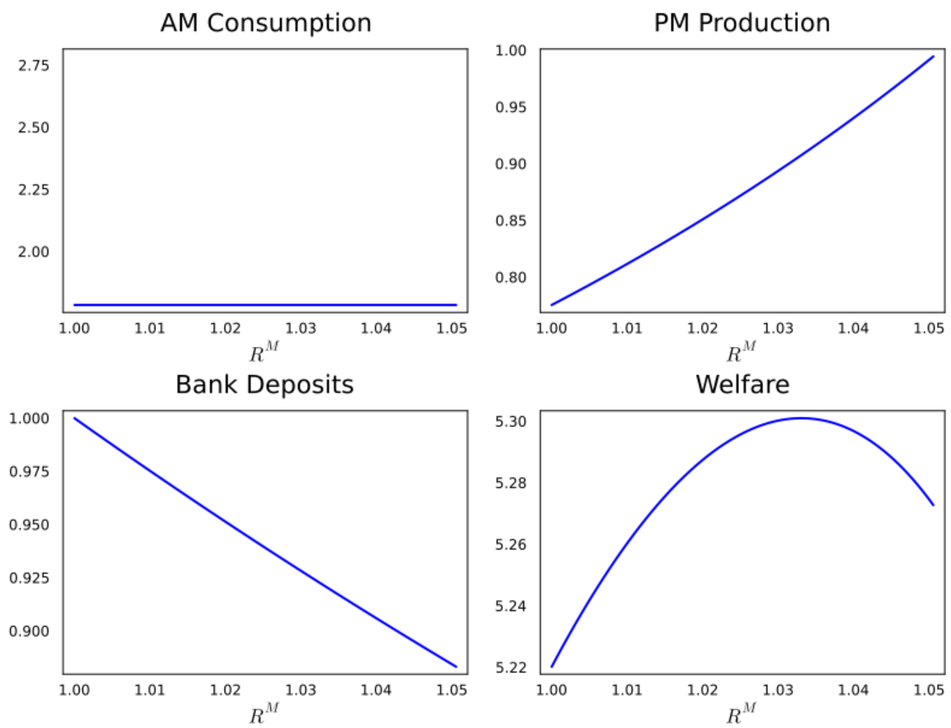

Figure 4: Effects of Interest-bearing CBDC through R^M

AM consumption remains constant across all values of R^M. The calibrated value is

$X = B(1-\tau_h)/A = 1.785$, and this level is invariant to CBDC policy: the household's AM optimality condition with $U(x) = B\log(x+\varepsilon)$ pins X to a function of (B, τ_h, A) alone, none of which is a CBDC policy lever. This invariance is consistent with the theoretical prediction that AM consumption is governed by the labor-leisure tradeoff and is largely insulated from monetary policy.

PM production responds substantially to CBDC policy. Raising R^M from 1 to 1.02 increases

Variables	$R^M = 1.00$	$R^M = 1.02$	$R^M = 1.05$
Levels			
$\delta = R^M/\mu$	0.980	1.000	1.029
PM production y	0.776	0.851	0.992
Investment / Deposits k	1.000	0.951	0.884
AM consumption X	1.785	1.785	1.785
Welfare	5.220	5.288	5.275
Changes relative to $R^M = 1.00$			
Δ Bank Deposits	—	−4.87%	−11.56%
Δ PM Production	—	+9.73%	+27.86%
Δ Welfare	—	+1.29%	+1.04%

Table 2: Counterfactual Effects of CBDC Policy

PM output from 0.776 to 0.851 (a 9.73% increase); raising R^M further to 1.05 increases PM output to 0.992 (a 27.86% increase over the benchmark). This expansion reflects the relaxation of the binding consumer debt-constraint as interest-bearing CBDC raises the real return on liquidity holdings. The PM market thus moves toward its first-best allocation as the CBDC rate rises within the admissible range. The magnitude of this response is driven by the calibrated PM utility curvature $\chi = 0.16$, which implies an elastic mapping from the policy ratio δ to PM output.

The model generates substantial bank disintermediation. Bank deposits fall from 1.000 at $R^M = 1$ to 0.951 at $R^M = 1.02$ (a 4.87% decline), to approximately 0.921 at the constrained welfare-maximizing $R^M \approx 1.034$ (a 7.9% decline), and further to 0.884 at $R^M = 1.05$ (an 11.56% decline). This disintermediation is the structural counterpart of Lemma 2: sustaining mixed CBDC–deposit equilibrium when CBDC pays interest requires bank deposits to offer a comparable real return, which through the bankers' first-order condition $f'(k) = R^D$ requires lower investment. The mechanism operates through the rate-of-return equality condition (29), reduced in steady state to $R^D = R^M$.[13] The magnitude of the deposit response is governed by the production curvature $\theta = 0.397$ through the closed form $k = (R^M)^{-1/\theta}$.

Despite this disintermediation, welfare effects are positive over most of the admissible range, with an interior welfare-maximizing value of R^M. Welfare rises from 5.220 at $R^M = 1$ to a peak

[13]This is assuming that bank deposits lose their illiquidity premium, that is, $f'(a) = 0$.

of 5.288 near $R^M = 1.02$ (a 1.29% increase) before declining slightly to 5.275 at $R^M = 1.05$ (a 1.04% increase over the benchmark). The interior peak is a quantitative refinement of the theoretical tradeoff identified: higher R^M relaxes the household debt constraint and improves PM efficiency, but also crowds out the bank deposits that fund productive investment. At low CBDC rates the PM efficiency channel dominates; at higher rates the disintermediation channel begins to bite. The optimum in my calibration sits at a CBDC nominal interest rate of approximately 2 percentage points above zero.

These results refine the policy tradeoff identified theoretically. Active CBDC policies with positive nominal interest rates can improve welfare even in the presence of disintermediation, but only up to a point. Beyond approximately $R^M = 1.02$, further increases in the CBDC interest rate produce additional disintermediation that begins to outweigh the PM efficiency gains. The constrained welfare-maximizing policy in this model is therefore an active but moderate CBDC interest rate, with the precise level disciplined by the calibrated curvature parameters governing the relative strength of the two channels.

7.3 Welfare costs of active and passive policies

To quantify the welfare implications of CBDC policy more precisely, I follow Wang (2016) and compute the consumption-equivalent welfare cost of moving from the $\delta = 1$ benchmark[14] to active policies with $\delta > 1$. For any policy δ, total welfare in steady state is

$$\text{Wel}(\delta) \;=\; (1-\beta)^{-1}\big[U(X(\delta)) - A\,H(\delta) + \pi\left(u(y(\delta)) - g(y(\delta))\right)\big]. \tag{62}$$

The consumption-equivalent welfare cost is the value $1 - \Delta_1$ solving $\text{Wel}(\delta) = \text{Wel}_{\Delta_1}(1)$, where

$$\text{Wel}_{\Delta_1}(1) \;=\; (1-\beta)^{-1}\big[U(\Delta_1\,X(1)) - A\,H(1) + \pi\left(u(y(1)) - g(y(1))\right)\big] \tag{63}$$

scales AM consumption at the passive baseline by the factor Δ_1 while holding labor and PM allocations at their baseline levels. Negative values of $1 - \Delta_1$ indicate that the active policy is welfare-improving relative to the passive baseline; positive values indicate a welfare cost.

Figure 5 plots the resulting welfare-cost curve over $\delta \in [1, 1.05]$. The welfare-cost curve confirms the aforementioned interior-optimum finding. Table 3 reports the values at representative policy points along with the constrained welfare-maximizing δ identified on the grid. The constrained welfare-maximizing policy on the grid is $\delta^* \approx 1.013$, corresponding to a CBDC

[14]Strictly speaking, the Section 4 passive policy ($\mu = 1$, $R^M = 1$) is off this restricted menu because μ is fixed at 1.02; $\delta = 1$ here corresponds to zero real interest on CBDC ($R^M = \mu$), not the formal passive policy.

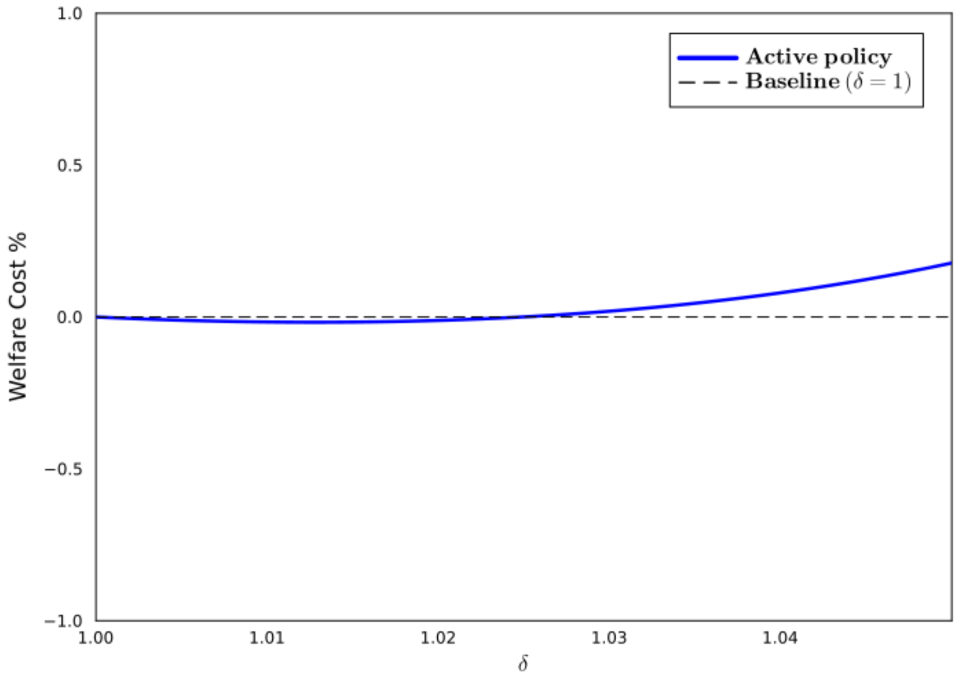

Figure 5: The Welfare Cost of CBDC

Variables	$\delta = 1.00$	$\delta = 1.013^*$	$\delta = 1.02$	$\delta = 1.05$
$R^M = \mu\delta$	1.020	1.034	1.040	1.071
CBDC nominal rate	2.0%	3.4%	4.0%	7.1%
Δ_1	1.000000	1.000173	1.000113	0.998223
Welfare cost $1 - \Delta_1$ (%)	0.000	−0.017	−0.011	+0.178
Welfare Wel(δ)	5.288	5.296	5.296	5.148

* Welfare-maximizing δ on the grid.

Table 3: Consumption-equivalent Welfare Costs across CBDC Policies

nominal interest rate of approximately 3.4 percentage points. At this point, the household would be indifferent between the active policy and a counterfactual passive policy in which AM consumption were scaled up by 0.017%. For δ above approximately 1.027, the welfare cost becomes positive, meaning the active policy is welfare-reducing relative to the passive baseline. At $\delta = 1.05$ the welfare cost is 0.178%.

At the constrained welfare-maximizing $\delta^* \approx 1.013$, the CBDC fee constraint (48) is slack: the seigniorage and labor income tax revenue available at the calibrated (μ, τ_h) comfortably cover the CBDC interest obligation, so the policy is incentive-feasible without requiring $\tau > 0$. The constraint remains slack throughout the admissible range $R^M \in [1, 1.05]$. At the standard Friedman rule $\delta = 1/\beta$, by contrast, the right-hand side $(\delta - 1)M$ exceeds the revenue available from μ and τ_h alone.

The consumption-equivalent welfare gain at the optimum is small in absolute magnitude. This conservative magnitude reflects a specific feature of the calibrated model: AM consumption

$X = B(1-\tau_h)/A$ is invariant to CBDC policy because the AM optimality condition under $U(x) = B\log(x+\varepsilon)$ pins X to parameters that are themselves fixed across policies. The consumption-equivalent metric folds the entire welfare difference into a scalar applied to AM consumption, which is a flat variable across δ in this model. The welfare effect itself, however, operates through PM trade and labor effort: both PM consumption y and PM disutility $g(y)$ rise with δ, producing partially offsetting changes in the PM welfare term $\pi[u(y) - g(y)]$, while net labor effort $H = X + k - f(k)$ falls as investment declines.

In terms of the level of total welfare, the gain from moving from $\delta = 1$ to δ^* is $\text{Wel}(\delta^*) - \text{Wel}(1) = 0.008$, or approximately 0.16% of baseline welfare. This is the natural welfare-change comparison and is the magnitude implicit in the welfare panels of Figures 3 and 4. The consumption-equivalent metric of 0.017% is mechanically smaller because it constrains all the welfare gain to be expressed through a single channel (AM consumption) that the model holds constant. Both metrics describe the same underlying allocation differences, and both confirm that the welfare gains from moderate interest-bearing CBDC are modest in magnitude but real in sign.

Based on these results, the design of an interest-bearing CBDC policy should aim for a positive but moderate nominal interest rate. Taking μ and τ_h as given at their calibration values, the constrained welfare-maximizing CBDC nominal rate is approximately 3.4%, with the welfare cost turning positive beyond approximately 2.7% above $R^M = \mu$. While these magnitudes are small, the qualitative finding is robust across the calibration: active CBDC policy can improve welfare even when it crowds out bank deposits, but only up to a disintermediation-bounded interior optimum. This finding identifies a key policy-design tradeoff that the existing CBDC literature has emphasized qualitatively but quantified under varying assumptions; the calibration here provides one disciplined quantitative anchor.

7.4 Patient-regime benchmark

The quantitative analysis so far has focused on the calibrated US economy, which was placed in the impatient region $A < A_0$ where binding debt-constraints make active CBDC policy welfare-improving. Proposition 1 establishes that the model also admits a patient region, $A \geq A_0$, in which a passive policy $(\delta, \tau, \tau_h) = (1, 0, 0)$ with non-interest-bearing CBDC implements the first-best allocation. This subsection illustrates that regime with the calibrated parameters.

In a patient economy with $A \geq A_0$, the consumer debt-constraint is slack. PM output attains its first-best level y^*, satisfying $u'(y^*) = g'(y^*)$, which at the calibrated curvature gives $y^* = 1.000$. Investment attains $k^* = \beta^{1/\theta} = 0.926$, satisfying $\beta f'(k^*) = 1$. By Lemma 1, y^* and k^* are independent of A; only first-best AM consumption X^* depends on A. Asset prices are at their fundamental levels, $\phi_1 = \phi_1^*$ and $\phi_2 = \phi_2^*$, so there is no liquidity premium and no disintermediation: bank deposits are held at the level that funds first-best investment. Computing the threshold A_0 from (50) at the calibrated $(\beta, \pi, \chi, \theta)$ and the wage normalization $w = 1$ yields $A_0 \approx [value]$,[15] so any $A \geq A_0$ places the economy in the patient regime; the calibrated value $A = 1$ falls below this threshold, confirming that the patient regime is not a corner case and that the boundary between the two regimes is pinned down by the model's preference parameters.

To make the regime contrast explicit, it is useful to compare the first-best allocation that passive policy achieves in the patient economy with the constrained allocation that would arise in the *same* economy if the debt-constraints were binding. The comparison is within-economy: it holds the preference parameter A fixed and varies only whether the constraint binds, which is the comparison Propositions 1 speaks to. In the patient regime, the policy $(\delta, \tau, \tau_h) = (1, 0, 0)$ delivers $y = y^* = 1.000$, $k = k^* = 0.926$, and $\phi_1 = \phi_1^*$ at its fundamental level; in the constrained counterpart, the same economy would require active policy and yield $y < y^*$, $k < k^*$, and a strict liquidity premium $\phi_1 > \phi_1^*$.

The calibrated economy has $A = 1$ and lies in the impatient region $A < A_0$; Section 7.2 showed that it requires an active CBDC policy to relax binding debt-constraints and that even the constrained welfare-maximizing active policy leaves PM output below y^*. A sufficiently patient economy, by contrast, reaches the first-best allocation under a passive policy alone, with no interest-bearing CBDC and no bank disintermediation. Welfare levels are not compared across the two economies, since A is a preference parameter and by Lemma 1 governs first-best AM consumption directly; the substantive contrast is that the policy required for first-best implementation differs across the two regimes. This is the quantitative counterpart of the central theoretical message: whether interest-bearing CBDC is needed depends on how patient an economy is, and in sufficiently patient economies a passive policy with non-interest-bearing CBDC does not crowd out bank deposits.

8 Conclusion

Many policymakers are debating whether CBDCs should be interest-bearing and universally accessible. In this study, I extend a fairly standard monetary model in the New Monetarist tradition to explore the optimal design and implementation of central bank digital currencies in an economy where both CBDCs and private bank deposits coexist as competing payment instruments. The main findings suggest that the welfare consequences of CBDCs and the policies required for first-best implementation depend on specific parameters. Another issue I address is whether private intermediaries should be left to provide exchange media to facilitate trade when commitment and record-keeping are limited. There is no obvious answer to this, as it depends

[15] Evaluated at $y^* = 1$ and $k^* = \beta^{1/\theta} = 0.926$, with $v_1 M + \phi_1^* D$ pinned by the rate-of-return equality condition (29)) at $R^M = 1$. The wage normalization $w = 1$ follows the calibration convention adopted in Section 7.1.

on the state of the economy.

In sufficiently patient economies, a passive monetary policy with constant money supply and non-interest bearing CBDC can achieve the first-best allocation. In these economies, CBDC does not disintermediate or crowd out bank deposits, as banks are willing to issue deposits and pay sufficiently high interest rates. Asset prices reflect their fundamental values and there are no liquidity shortages.

Conversely, in more impatient economies, active policies with positive inflation and nominal interest rates are necessary to implement first-best allocations when CBDC competes with bank deposits. In these economies, binding debt constraints lead to liquidity shortages, causing banks to issue fewer deposits at excessively high interest rates. Consequently, asset prices become overvalued, reflecting liquidity premia. To restore efficiency, the government must use a combination of seigniorage revenue, labor income taxes, and CBDC fees to incentivize households to increase production. Importantly, interest-bearing CBDC in these environments could potentially crowd out bank deposits if the interest rate on CBDC is too high relative to deposit rates. The disintermediation of banks then comes at the expense of improving efficiency. This tradeoff is one that central banks should consider when designing policies regarding CBDCs that are incentive-feasible

The findings of this paper contribute to the growing literature on CBDCs and provide valuable insights for policymakers considering the introduction of CBDCs. The results highlight the importance of carefully designing CBDC policies based on the specific characteristics and preferences of the economy to maximize welfare and minimize potential risks to financial intermediation. Future research could extend this framework to incorporate additional features such as heterogeneous agents, financial frictions in the banking sector such as bank market power and pledgeability constraints, and monetary policy transmission channels to further deepen our understanding of the macroeconomic implications of CBDCs.

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Appendix

Proof of Lemma 3. To derive $\tau^*(\delta, R^D, A)$, set $A = wg'(y^*)y^* \left\{\beta\left[\delta v_1 M + (R^D + f'(a))\phi_1 D\right]\right\}^{-1}$ in (50) to obtain

$$\tau_h = 1 - \frac{A\beta[\delta v_1 M + (R^D + f'(a))\phi_1^* D]}{wg'(y^*)y^*}.$$

Use the government budget constraint (31) to substitute out τ_h to obtain

$$\tau = (1-\pi)^{-1}\left\{(\delta - 1)M + \frac{\beta AH[\delta v_1 M + (R^D + f'(a)\phi_1^*)D]}{g'(y^*)y^*} - wH\right\}.$$

Use (40) to substitute ϕ_1^* and simplify to obtain (57). Differentiating (57) with respect to A leads to

$$\frac{\partial \tau^*(\delta, R^D, A)}{\partial A} = \frac{\beta H(1-\pi)^{-1}[\delta v_1 M(1-\beta R^D) + R^D D]}{(1-\beta R^D)g'(y^*)y^*} > 0.$$

Now, differentiating (57) with respect to δ leads to

$$\frac{\partial \tau^*(\delta, R^D, A)}{\partial \delta} = (1-\pi)^{-1}M + \frac{\beta AH(1-\beta R^D)(1-\pi)^{-1}v_1 M}{(1-\beta R^D)g'(y^*)y^*} > 0.$$

Finally, differentiating (57) with respect to R^D gives us

$$\begin{aligned}\frac{\partial \tau^*(\delta, R^D, A)}{\partial R^D} = &\frac{(1-\beta R^D)g'(y^*)y^*[\beta AH(1-\pi)^{-1}D - \beta^2 A(1-\pi)^{-1}\delta v_1 M]}{\left[(1-\beta R^D)g'(y^*)y^*\right]^2} \\ &+ \frac{\beta^2 AH(1-\pi)^{-1}\left[\delta v_1 M(1-\beta R^D) + R^D D\right]g'(y^*)y^*}{\left[(1-\beta R^D)g'(y^*)y^*\right]^2} \lesseqgtr 0.\end{aligned}$$

■

Lemma 4 (Sequential rationality) *Under an active policy with $f(k) = k^{1-\theta}/(1-\theta)$ and PM disutility $g(y) = y^2/2$, a sufficient condition for sequential rationality (49) to hold at the equilibrium policy is*

$$\pi\frac{u(y^*)}{(y^*)^2} + \beta\left[\pi\mu\alpha + (1-2\pi)\,\kappa\,(1-\alpha)\right] \geq \frac{1}{\beta^2}\left[\frac{\alpha}{\delta} + \frac{1-\alpha}{R^D + f'(D)}\right], \qquad (64)$$

where $\alpha \equiv v_2 M/y^* \in [0,1]$ is the equilibrium CBDC share of PM consumption and

$$\kappa(\theta, \beta R^D) \equiv \frac{1-\theta\beta R^D}{1-\theta}. \qquad (65)$$

In the limit $\tau_h \to 0$, $w = 1$, and $\theta \to 0$, condition (64) at the Friedman-rule policy ($\mu = \beta$, $\delta = 1/\beta$, $\beta \to 1$) collapses to

$$\frac{u(y^*)}{(y^*)^2} \;\geq\; 2,$$

which is the natural analog of Andolfatto et al. (2016) Lemma 4 under quadratic producer disutility.

Proof of Lemma 4. The proof follows the structure of Andolfatto et al. (2016) Lemma 4, adapted to the two-asset framework with a labor-income tax. Following the one-time deviation principle, I compare the equilibrium lifetime utility $W(0,0)$ of an agent who enters the AM with zero balances and rebalances to (M, D) against the lifetime utility $\widehat{W}(0,0)$ of a deviator who chooses not to rebalance.

Step 1: Equilibrium and deviator payoffs.

A consumer entering the AM with $(z, a) = (0, 0)$ has $\sigma = 0$ trivially. Substituting (10) for h and evaluating at the equilibrium AM consumption $x = x^*$,

$$W(0,0) \;=\; U(x^*) - \frac{A\,[x^* + v_1 M + \phi_1 D]}{(1-\tau_h)\,w} \;+\; \beta V(M,D), \tag{66}$$

where

$$V(M,D) \;=\; \pi\,[u(c) + \beta W(0,0)] \;+\; \pi\,[-g(y) + \beta W(2M,2D)] \;+\; (1-2\pi)\,\beta W(M,D), \tag{67}$$

with $c = y = v_2 M + \phi_2 D$ in equilibrium.

The deviator chooses $(m, d) = (0, 0)$ and $x = x^*$, so

$$\widehat{W}(0,0) \;=\; U(x^*) - \frac{A\,x^*}{(1-\tau_h)\,w} \;+\; \beta\,\widehat{V}(0,0). \tag{68}$$

With probability π the deviator is assigned the consumer role but cannot consume (the debt-constraint binds at $(0, 0)$), so he is effectively idle; with probability π he is a producer who produces y and accepts (M, D) from his trading partner; with probability $(1 - 2\pi)$ he is an idler. Thus,

$$\widehat{V}(0,0) \;=\; \pi\,[-g(y) + \beta W(M,D)] \;+\; (1-\pi)\,\beta W(0,0). \tag{69}$$

Step 2: Sequential rationality reduces to a difference of PM values.

The condition $W(0,0) \geq \widehat{W}(0,0)$ reduces (the $U(x^*)$ and $Ax^*/[(1-\tau_h)w]$ terms cancel) to

$$\beta\left[V(M,D) - \widehat{V}(0,0)\right] \;\geq\; \frac{A\,[v_1 M + \phi_1 D]}{(1-\tau_h)\,w}. \tag{70}$$

Subtracting (the $-\pi g(y)$ terms cancel) yields

$$V(M,D)-\widehat{V}(0,0) \;=\; \pi u(y^*)-(1-2\pi)\beta W(0,0)+\pi\beta W(2M,2D)+(1-3\pi)\beta W(M,D). \quad (71)$$

Step 3: Linearity of W in entry balances.

For an agent in the AM with (z,a) who chooses $\sigma = 1$ (the equilibrium choice for idlers and producers), the AM budget constraint (10) combined with the symmetry of the equilibrium portfolio $(m,d) = (M,D)$ implies

$$W(M,D) \;=\; W(0,0)+\frac{A\,[R^M v_1 M-\tau]}{(1-\tau_h)\,w}, \qquad W(2M,2D) \;=\; W(0,0)+\frac{A\,[2R^M v_1 M-\tau]}{(1-\tau_h)\,w}. \quad (72)$$

Substituting into (71), the coefficient of $W(0,0)$ is $-(1-2\pi)+\pi+(1-3\pi) = 0$, so the $W(0,0)$ terms cancel. Collecting the coefficients of $R^M v_1 M$ and τ,

$$V(M,D)-\widehat{V}(0,0) \;=\; \pi u(y^*)+\frac{\beta A}{(1-\tau_h)\,w}\big\{(1-\pi)\,R^M v_1 M-(1-2\pi)\,\tau\big\}. \quad (73)$$

Substituting (73) into (70),

$$\beta\pi u(y^*)+\frac{\beta^2 A}{(1-\tau_h)\,w}\big\{(1-\pi)\,R^M v_1 M-(1-2\pi)\,\tau\big\} \;\geq\; \frac{A\,[v_1 M+\phi_1 D]}{(1-\tau_h)\,w}. \quad (74)$$

Step 4: Worst case for τ.

Since $\pi \leq 1/2$, the coefficient of τ in (74) is non-positive: increasing τ tightens the constraint. The hardest case for sequential rationality occurs when τ saturates the idler incentive constraint (16),

$$\tau \;=\; R^M v_1 M-\phi_1\big(R^D D+f(D)\big). \quad (75)$$

Substituting and simplifying,

$$(1-\pi)\,R^M v_1 M-(1-2\pi)\big[R^M v_1 M-\phi_1\big(R^D D+f(D)\big)\big] \;=\; \pi\,R^M v_1 M+(1-2\pi)\,\phi_1\big(R^D D+f(D)\big). \quad (76)$$

Equation (74) becomes

$$\beta\pi u(y^*) + \frac{\beta^2 A}{(1-\tau_h)\,w}\big\{\pi R^M v_1 M + (1-2\pi)\,\phi_1\big(R^D D + f(D)\big)\big\} \;\geq\; \frac{A\,[v_1 M + \phi_1 D]}{(1-\tau_h)\,w}. \tag{77}$$

Step 5: Curvature correction for $\phi_1(R^D D + f(D))$.

With $f(k) = k^{1-\theta}/(1-\theta)$, one has $f(D) = Df'(D)/(1-\theta)$, so

$$\phi_1\big(R^D D + f(D)\big) \;=\; \phi_1 D\Big[R^D + \tfrac{f'(D)}{1-\theta}\Big] \;=\; \phi_1 D\,(R^D + f'(D)) + \tfrac{\theta}{1-\theta}\phi_1 D f'(D). \tag{78}$$

By the banker's first-order condition (9), $\phi_1 f'(D) = R^D$, so $\phi_1 D f'(D) = R^D D$. By the deposit asset-pricing identity (40), $\phi_1 = \beta R^D/(1-\beta R^D)$, which gives $1+\phi_1 = 1/(1-\beta R^D)$ and hence $R^D D = \phi_1 D/[\beta(1+\phi_1)] = \phi_1 D\,(1-\beta R^D)/\beta$. Substituting and using

$$\phi_1 D(R^D + f'(D)) = R^D D + \phi_1 D f'(D) = R^D D(1+\phi_1) = R^D D/(1-\beta R^D), \tag{79}$$

one obtains

$$\phi_1\big(R^D D + f(D)\big) \;=\; \kappa(\theta, \beta R^D)\,\phi_1 D\,(R^D + f'(D)), \qquad \kappa \;=\; \frac{1-\theta\,\beta R^D}{1-\theta}. \tag{80}$$

Step 6: Substitute the producer's PM FOCs.

From the producer's PM first-order conditions (25)–(26) at the first-best ($g'(y^*) = u'(y^*) = y^*$ under quadratic g) and stationarity ($v_1^+ M = v_1 M/\mu$, $\phi_1^+ = \phi_1$),

$$R^M v_1 M \;=\; \frac{\mu\, v_2 M\, u'(y^*)}{\beta\, U'(x^*)}, \qquad (R^D + f'(D))\,\phi_1 D \;=\; \frac{\phi_2 D\, u'(y^*)}{\beta\, U'(x^*)}, \tag{81}$$

and consequently

$$v_1 M \;=\; \frac{v_2 M\, u'(y^*)}{\beta\delta\, U'(x^*)}, \qquad \phi_1 D \;=\; \frac{\phi_2 D\, u'(y^*)}{\beta(R^D + f'(D))\, U'(x^*)}. \tag{82}$$

Substituting these and (80) into (77), and using $A/[(1-\tau_h)w] = U'(x^*)$,

$$\frac{\beta\pi u(y^*)}{U'(x^*)} + \frac{\beta\, u'(y^*)}{U'(x^*)}\{\pi\mu\, v_2 M + (1-2\pi)\kappa\,\phi_2 D\} \;\geq\; \frac{u'(y^*)}{\beta\, U'(x^*)}\left[\frac{v_2 M}{\delta} + \frac{\phi_2 D}{R^D + f'(D)}\right]. \tag{83}$$

Step 7: Normalize by y^* and PM market-clearing.

PM market-clearing at the first-best gives $v_2 M + \phi_2 D = y^*$. Set $\alpha = v_2 M/y^* \in [0,1]$ and $1-\alpha = \phi_2 D/y^*$. Multiplying through by $U'(x^*)/u'(y^*)$ and using $u'(y^*) = y^*$ under quadratic g,

$$\beta\pi \frac{u(y^*)}{y^*} + \beta y^* \{\pi\mu\alpha + (1-2\pi)\kappa(1-\alpha)\} \geq \frac{1}{\beta}\left[\frac{\alpha y^*}{\delta} + \frac{(1-\alpha) y^*}{R^D + f'(D)}\right]. \tag{84}$$

Dividing by y^* yields the sufficient condition (64).

Step 8: Limit case under the calibration's functional forms.

At $\tau_h = 0$ and $w = 1$, the labor-tax wedge collapses: $U'(x^*) = A$. As $\theta \to 0$, the curvature factor satisfies $\kappa \to 1$. At the Friedman-rule policy, $\mu = \beta$ and $\delta = 1/\beta$, so $R^M = 1$ and all wealth shifts to deposits: $\alpha \to 0$. The deposit asset-pricing identity then gives $\beta(R^D + f'(D)) \to 1$, so $R^D + f'(D) \to 1/\beta$. Substituting these limits into (64),

$$\pi \frac{u(y^*)}{(y^*)^2} + \beta(1-2\pi) \geq \frac{1}{\beta}.$$

At $\beta \to 1$, this rearranges to

$$\pi \frac{u(y^*)}{(y^*)^2} \geq 1 - (1-2\pi) = 2\pi, \quad \text{i.e.,} \quad \frac{u(y^*)}{(y^*)^2} \geq 2.$$

This is the analog of Andolfatto et al. (2016)'s Lemma 4 condition under quadratic producer disutility. ■